INFORMATION PAGE OF DOCTORAL THESIS

Name of thesis: The injectivity, projectivity of modules and some related classes of rings

Major: Algebra and Number Theory

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Abstract

This thesis studies pseudo-projective modules to characterize classical rings and nilpotent-invariant modules over a formal triangular matrix ring. We investigate rings in which all right ideals are automorphism-invariant and do not contain an infinite set of orthogonal idempotent elements. We study rings in which every ideal is nilpotent-invariant. This thesis has three chapters. In Chapter 1, we review the concepts and background results for Chapters 2 and 3. In Chapter 2, we characterize right perfect rings, semiperfect rings, right semihereditary rings, right hereditary rings and their extensions with the pseudo-projective cover and pseudo-projectivity of modules. We present some characterizations of semilocal rings and Artinian principal ideal rings via pseudo-projective modules. The main result of this chapter shows that the class of pseudo-projective modules coincides with the class of pseudo-injective modules. In Chapter 3, we study the class of right I-finite a-rings and generalize a-rings to rings in which every right ideal is nilpotent-invariant, called right $\mathfrak{n}$-rings. The main result of this chapter is the determination of the structure of these classes of rings. Every right I-finite a-ring is a direct sum of a semisimple Artinian ring and a basic semiperfect ring. We also prove that every indecomposable non-simple right I-finite a-ring with nontrivial idempotents is a quasi Frobenius ring. We show that a right $\mathfrak{n}$-ring has a ring decomposition into the direct sum of a square-full semisimple Artinian ring and a right square-free ring. We conclude that every right $\mathfrak{n}$-ring satisfies the internal cancellation property and is stably-finite. Furthermore, every regular right $\mathfrak{n}$-ring with zero right socle is strongly regular. Besides, we also provide characterizations of nilpotent-invariant modules over formal triangular matrix rings. Let $K = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$ be a formal triangular matrix ring, where R and S are rings, M is an R-S-bimodule. If (X, Y, f) is a nilpotent-invariant right K-module then Y is a nilpotent-invariant right S-module and $H = \{x \in X \mid f(x \otimes m) = 0, \forall m \in M\}$ is a nilpotent-invariant right R-module. From the obtained results, this thesis plays an important role in the study of ring and module theory and contributes to enriching the characterizations of classical rings. Generalizations of right a-rings have led to a new class, called right $\mathfrak{n}$-rings, opening up another research direction for ring and module theory researchers.

Key words: pseudo-projective module, nilpotent-invariant module, quasi Frobenius ring, a-ring, $\mathfrak{n}$-ring